Experimental and Numerical Analysis of Free Vibration and Dynamic Behaviour of Laminated Composite Shallow Shell under Hygrothermal Condition

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Experimental & Numerical Analysis of Free Vibration & Dynamic Behaviour of Laminated Composite Shallow Shell under Hygrothermal Condition

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by

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We the below signed, after checking the thesis mentioned above and the official record book (s) of the student, hereby state our approval of the thesis submitted in partial fulfillment of the requirements of the degree of Masters of Technology in Machine Design and Analysis at National Institute of Technology Rourkela. We are satisfied with the volume, quality, correctness, and originality of the work.

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Supervisor's Certificate

This is to certify that the work presented in this thesis entitled **"Experimental & Numerical Analysis of Free Vibration & Dynamic Behavior of Laminated Composite Shallow Shell under Hygrothermal condition"** by **"Rupavath Srikanth"**, Roll Number **214ME1290**, is a record of original research carried out by him under my supervision and guidance in partial fulfillment of the requirements of the degree of **Masters of Technology in Mechanical Engineering**. Neither this thesis nor any part of it has been submitted for any degree or diploma to any institute or university in India or abroad.

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Declaration of Originality

I, **Rupavath Srikanth**, Roll Number **214ME1290** hereby declare that this thesis entitled **"Experimental & Numerical Analysis of Free Vibration & Dynamic Behavior of Laminated Composite Shallow Shell under Hygrothermal condition"** represents my original work carried out as a postgraduate student of NIT Rourkela and, to the best of my knowledge, it contains no material previously published or written by another person, nor any material presented for the award of any other degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the thesis. Works of other authors cited in this dissertation have been duly acknowledged under the section "References". I have also submitted my original research records to the scrutiny committee for evaluation of my thesis.

I am fully aware that in case of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present thesis.

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Abstract

Laminated composite materials are extensively used in defense, aerospace, aircraft industry and in advanced sports industry because of high stiffness to weight ratio, high strength to weight ratio, light weight, and low cost compared to conventional materials. During the service of these materials, it is affected by the temperature and moisture. The study deals with the numerical and experimental investigations of free vibration and dynamic behavior of composite shell panel subjected to the hygrothermal environment. A mathematical model of the laminated composite cylindrical shell panel is developed using the first order shear deformation theory under different temperatures and moisture concentrations. Based on the developed mathematical model a computer program is developed in the ANSYS environment using finite element method. The convergence test is performed for the presently developed model and validated by comparing the responses with the results available in the published literature. For the experimental analysis laminated composite, cylindrical shell is fabricated using hand layup method. The natural frequency of the composite panel is computed experimentally for different aspect ratios with the help of the vibration analyzer (NI-cDAQ) in conjunction with LAB-VIEW program and the result is compared with the simulation results. The effect of different parameters like aspect ratio, thickness ratio, and support condition and convergence study are analyzed using presently developed model for both free and dynamic analysis and discussed in detail.

Key words: Hygrothermal, ANSYS, LAB-VIEW, Vibration, CDAQ

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Nomenclature

u_0, v_0, w_0	Mid plane Displacement about x, y, z direction
θ_x, θ_y	Rotation perpendicular to y *and* x axis
a, b *and* h	Length, breadth and thickness of laminated composite cylindrical shell panel
R	Radius of curvature of cylindrical shell panel
E_1, E_2 *and* E_3	Young's modulus
G_{12}, G_{23} *and* G_{13}	Shear modulus
$\upsilon_{12}, \upsilon_{23}$ *and* υ_{13}	Poison's ratios
ρ	Density of the martial
$[M]$	Global mass matrix
$[K]$	Global stiffness matrix
$\{u\}$	Displacement vector
$\{\ddot{u}\}$	Acceleration vector
$\{F\}$	Global load vector

Chapter 1

Introduction

1.1 Overview

Composite is a material that are made by combining two or more constituent material and the properties of the both the materials different from each other. When the materials are combined they form a new material witch have different characteristic from the both parent materials. One constituent material is called reinforcing phase and the other constituent material is called matrix. Matrix phase material is in the continuous form and the reinforcing phase may be in the form of particle, fiber or flakes.

These materials are preferred as compare to the conventional material for some reasons such as: low cost, low weight and stronger. Some of the examples of the composite are concrete reinforced with the steel, fiber reinforced polymers, metal composite and epoxy reinforced with the graphite fibers. Some naturally found composites are wood, human bones and teeth, pearl related shell structure.

Advanced composites are composites that are determined by high strength of the fiber and high stiffness and high modulus of elasticity compare to the composite materials. These advanced composites are used in aerospace industry applications. Advanced composites are extensively used in defiance, aerospace, aircraft industry and in advanced sports industry.

Composite materials mainly classified in to following type

- Fibrous composite material : contains fiber in matrix
- Laminated composite material : contains layers of various materials
- Particulate composite material : combining particles in matrix
- Combination of all or some of above

1.2 Laminated composite material

Laminated composite material are combination of different fibrous composite material which are joined together to form a material with required material properties. By lamination the properties that can be achieved are strength, stiffness, thermal resistance, low weight, wear resistance, corrosion resistance, and acoustic insulation.

The each layer consist of high modulus and high strength fibers in metallic, polymeric, ceramic matrix materials. Some of the fibers used are glass, boron, graphite and silicon carbide and typical matrix materials are epoxy, aluminum, polyimides, titanium and alumina. The individual layer may be some times orthotropic, isotropic, and anisotropic

1.3 Mechanical behaviour of composite material

Composite materials have numerous mechanical behavior characteristics. The mechanical behavior of the composite material qualities are not the same as traditional building materials. Traditional treatment is a combination of several properties; others absolutely need a new strategy and a new investigative and trial.

Most of the common engineering materials are homogeneous and isotropic in nature. A homogeneous nature of a body is exhibits same elastic properties at any point in a given direction, i.e., the elastic properties are independent of point. A material is said to be isotropic when it exhibits same elastic properties in any direction at a given point, i.e., the properties are independent of direction at a particular point in the body. When a temperature gradient in material properties under temperature-dependent isotropic bodies is not homogeneous, but is still isotropic. Composite materials are most often both in inhomogeneous nature and non-isotropic in nature. An inhomogeneous body has same elastic properties on the body, i.e., the elastic properties dependent of point in the body.

Orthotropic materials are the materials which exhibit different elastic properties in its orthogonal direction, i.e., it has different elastic properties in orthogonal directions. And it has at least two plane of orthogonal symmetry, i.e., the material property of the martials are independent of orthogonal directions. An isotropic material has different elastic properties at a point in all different directions of the body. There will not be any planes of symmetry of a material properties exist. The properties are dependent on the direction at a different point in the body.

Because of the composite materials heterogeneous nature, they are studied by two methods points of view those are micromechanics approach and macro mechanics approach. Micromechanics method is to investigate the behavior of composite materials where the interaction of the constituent materials is studied on microscopic scale to find their effect on the properties of the composite material. Macro mechanics method is to investigate the behavior of the composite material where the material is believed to be homogeneous material and the effects of the constituent materials are found as the averaged apparent macroscopic properties of the composite material.

1.4 Vibration of structures

In automobile, aerospace and naval industries vibrations of the components plays an essential parameter for the design and performance of the structure. Vibration is the mechanical occurrence where about the equilibrium point the oscillation occurs. This oscillation can be random or periodic. The vibration which occurs can be desirable or undesirable. Because of this vibration the life of the structure decreases due to the fatigue stresses in the structure increases. The efficiency of the machine affected because of the high level of noise produced. As a consequence of this to check the performance of structure vibration response are computed.

An exciter is required for the testing of vibration for large components and sub-assemblies. These exciters are portable, simple in operation and reliable. Mechanical vibration motion is produced in exciter to test the object. To check the dynamic characteristics vibration test is performed on structures. This exciter can produce time dependent excitation or harmonic force and displacement for a given range of frequencies. Exciters can be electrodynamics, mechanical or electro hydraulic.

Composite materials are usually applicable for bridges, buildings, structures like race car bodies, swimming pool panels, storage tanks, boar hulls. The advanced composites are mainly used in aircraft industries and aerospace industries. Often in the aerospace applications, aircraft, and defense applications the materials exposes to high temperatures and moisture conditions. The change in atmospheric conditions on the materials due to moisture absorptions and temperature has adversarial influence on the stiffness and the strength of the composite materials. These large series of practical application requires an essential understanding of vibration, dynamic stability and static characteristic under hygrothermal condition.

1.5 Finite Element Method and ANSYS

As the technology advances, the design process are nearing to the precision, with this finite element method is widely utilized and it is proficient to draw complex structures, this is highly reliable tool for designing any complex shape and structure. It plays the significant role in determining the response of the numerous parts, products, assemblies and subassemblies. These days, finite element method is widely utilized by all driven by commercial enterprises which are generally limited component prototyping technique to spare their season, and lowering the cost of the physical examination and the development of a speedier rate, and is used by more precise way. Presently in the market there are various enhanced finite element analysis tools are available and the ANSYS is one of them. This is acceptable to a lot of analyst and industries.

1.4.1 Steps for free vibration response in ANSYS

1. Define element type
2. Define material property
3. Create model area
4. Divide model into sections
5. Mesh the model
6. Define analysis type as modal analysis
7. Define boundary condition

8. Solve
9. Read results
10. Plot results

1.4.2 Steps for transient response in ANSYS

1. Define element type
2. Define material property
3. Create model area
4. Divide model into sections
5. Mesh the model
6. Define analysis type as transient
7. Define boundary condition
8. Define load
9. Define time at the end of the load step and time step size
10. Solve
11. From time history post processor define load at the node
12. Plot result

Chapter 2

Literature Review

2.1 Introduction

Composite materials are usually applicable for bridges, buildings, structures like race car bodies, swimming pool panels, storage tanks, boar hulls. The advanced composites are mainly used in aircraft industries and aerospace industries. Often in the aerospace applications, aircraft, and defense applications the materials exposes to high temperatures and moisture conditions. The change in atmospheric conditions on the materials due to moisture absorptions and temperature has adversarial influence on the stiffness and the strength of the composite materials. These large series of practical application requires an essential understanding of vibration, dynamic stability and static characteristic under hygrothermal condition. Though present study mainly focuses on the free vibration and transient analysis of composite laminate cylindrical shell under hygrothermal condition, some of the relevant researches on forced vibration, transient analysis on the composite plate and different radius of curvature shell panels and beams, etc. are also studied for its significance and completion.

Kundu *et al.* [1] investigated the nonlinear analysis of a doubly curved composite laminated shell by using finite element analysis under hygrothermal conditions i.e. the composite laminate is exposed to temperature and moisture conditions. Ganapathi *et al.* [2] presented the dynamic analysis of the cross ply laminated composite non circular cylindrical thick shells subjected mechanical/thermal load using higher order theory (HSDT). Also, the reactions of cylindrical shells are obtained by using direct time integration technique in conjunction with finite element approach. To study the hygrothermal effect on the thick laminate composite using the HSDT is developed by Patel *et al.* [3]. In this study, static and dynamic characteristic is done using COQUAD-8 isoperimetric higher order finite element. Parhi *et al.* [4] investigated the hygrothermal environment effect on the free vibration and dynamic analysis of the delaminated doubly curved laminated composite shell, in which the mathematical modulation was developed by using the first order shear deformation theory (FSDT). Effect of different temperature and moisture concentration the free vibration of laminated composite plate are investigated by Sairam and Sinha [5] and Patel *et al.* [6] using FSDT and HSDT. The analysis is done by finite element method using the quadratic iso-parametric element with clamped and simply supported boundary conditions. Lal *et al.* [7] investigated the impact of system properties of nonlinear laminated composite shell panel under hygrothermo mechanical loading condition due to the post buckling load. For the basic mathematical formulation von-Karman nonlinear kinematics and HSDT is used. Kundu and Han [8] investigated the vibration characteristic of hygrothermoelastic doubly curved laminated composite shell under pre and post buckling conditions. Huang and Tauchert [9] presented the large deformation behavior in non-uniform temperature loading condition of anti-symmetric angle ply curved panel. Whitney and Ashton [10] presented free vibration behavior of a composite laminate under hygrothermal condition based on classical laminate plate theory (CLPT) by using Ritz method. Panda and Singh [11] computed the nonlinear free vibration behavior of the post buckled state of single/doubly curved shell with uniform temperature field. Naidu and Sinha [12] investigated the nonlinear free vibration behavior of composite laminated shells under the hygrothermal environment by using finite element approach with Green Lagrange type nonlinearity. Ju *et al.* [13] presented the free vibration analysis of the composite laminated plates with different delamination by using the finite element formulation Mindlin plate theory. Marques and Creus [14] investigated the time dependent response of laminated polymer matrix composites under mechanical and hygrothermal loads. This analysis was based on the formulation of incremental total Lagrangian elements of the shells and plates. Whitney and Ashton [15] investigated that in addition to inducing residual stress, expansion strain can also affect the response characteristic of the composite. Ribeiro and Jansen [16] under the continuous action of the mechanical excitation and thermal field of geometrically nonlinear vibration of the linear elastic laminated composite shallow shell were analyzed. Nagai and Yamaki [17] theoretically analyzed the dynamic stability of the cylindrical shell in both periodic and static compressive forces under different four boundary conditions based on the Donnell equation. Galerkin procedure is used to reduce the problem into finite degree of system and the stability of this problem is studied by using Hsu's method. Bert and Birman [18] investigated analytically under axial loading condition the dynamic instability of circular cylindrical shell of finite length in simply supported boundary condition and FSDT is used for the mathematical modulations. Liew *et al.* [19] developed a numerical technique of a laminated composite cylindrical shell under the periodic and axial forces for the dynamic stability analysis by using mesh free kp-Ritz method. Effect of lamination schemes and boundary condition in the instability region is also in detail examined. Maleki *et al.* [20] presented the transient response of the composite laminate plates under different loading conditions like simply supported, clamped and free boundary conditions using generalized differential quadrature (GDQ) method based on FSDT. Reddy [21] developed the dynamic response of the symmetric cross ply laminated composite plate by resolving the equation of HSDT.

2.2 Knowledge gap

Form the above literature review it is observed that the free vibration and transient analysis of the laminated composite shell and plate are done under numerical simulation but best on the author's knowledge only few of work have been done in both numerical and experimental. So to bridge the gap in numerical simulation and experimental analysis the present work is focused to analyze the vibration and transient behavior using numerical simulation as well as experiment.

2.3 Motivation for present work

Composite laminated materials are extensively used in different industries like defense, aerospace, aircraft industry and in advanced sports industry because of the stiffness to weight ratio and the strength to weight ratio are high, low weight and low cost compared to conventional materials. Apart from these they also possess properties like corrosion resistance, chemical resistance, low coefficient of thermal expansion and also high elastic properties. The increase in the complex analysis of composite laminate is due to extensively used in the field like aerospace and aeronautical engineering. During the service life of these materials it will be exposed to the temperature and moisture and because of this these materials will be affected. Therefore a thorough study on composite laminate for the free vibration and transient response has to be examined under hygrothermal environment.

2.4 Objective and Scope of the present work

This work objective is to develop a finite element formulation experimental procedure to find natural frequency and transient response of a composite laminated cylindrical shell under hygrothermal loading condition. Glass epoxy composite cylindrical shell is used for this study.

- Developments of simulation model of laminated composite cylindrical shell based on first order deformation theory.
- Numerical analysis of vibration and dynamic behavior of cylindrical shell.
- Developments of experimental setup to conduct experimental vibration test on laminated composite cylindrical shell under cantilever type support condition.
- Investigation of vibration and dynamic behavior of fiberglass composite cylindrical shell experimentally.
- Validated experimental results with the finite element software package ANSYS.

Chapter 3

Theory and Formulation

3.1 Introduction

An eight node iso-parametric element as shown in the figure 3.1 is used for free vibration analysis of woven fiber composite plates under hygrothermal load. At each node six degree freedoms $u, v, w, \theta_x, \theta_y$ and θ_z considered. of

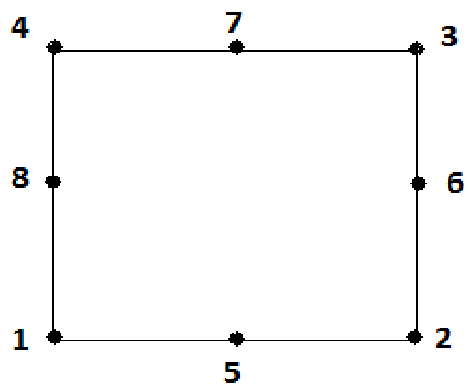

Figure 3.1 Eight node iso-parametric elemen

$$\left.\begin{aligned} u &= u_0 + z\theta_x \\ v &= v_0 + z\theta_y \\ w &= w_0 + z\theta_z \end{aligned}\right\} \tag{3.1}$$

u, v, w are displacement in *x, y, z* direction, u_0, v_0, w_0 are mid plane displacement about *x, y, z* direction, θ_x, θ_y are the rotation perpendicular to the y and x axis respectively

3.2 Strain-displacement relation

The generalized strain equation in terms of mid plane strain and curvature can be expressed by using Sanders first approximation theory.

$$\left.\begin{aligned}
\varepsilon_{xx} &= \left(\frac{\partial u}{\partial x} + \frac{w}{R_x}\right) \\
\varepsilon_{yy} &= \left(\frac{\partial v}{\partial y} + \frac{w}{R_y}\right) \\
\varepsilon_{zz} &= \left(\frac{\partial w}{\partial z}\right) \\
\gamma_{xy} &= \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}}\right) \\
\gamma_{yz} &= \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} - \frac{v}{R_y}\right) \\
\gamma_{zx} &= \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - \frac{u}{R_x}\right)
\end{aligned}\right\} \tag{3.2}$$

Strain expressed as

$$\varepsilon = \begin{bmatrix}\varepsilon_{xx} & \varepsilon_{yy} & \varepsilon_{zz} & \gamma_{xy} & \gamma_{yz} & \gamma_{zx}\end{bmatrix}^T \tag{3.3}$$

Using equation (3.1) and (3.2) the strain terms in the *x*, *y* and *z* direction are

$$\left(\varepsilon_{xx}\right) = \left(\frac{\partial u}{\partial x} + \frac{w}{R_x}\right) = \frac{\partial u_0}{\partial x} + z\frac{\partial \theta_x}{\partial x} + \frac{w_0}{R_x} + Z\frac{\theta_x}{R_x}$$

$$\left(\varepsilon_{xx}\right) = \left(\frac{\partial u_0}{\partial x} + \frac{w_0}{R_x}\right) + z\left(\frac{\partial \theta_x}{\partial x} + \frac{\theta_z}{R_x}\right)$$

$$\left(\varepsilon_{yy}\right) = \left(\frac{\partial v_0}{\partial y} + \frac{w_0}{R_y}\right) + z\left(\frac{\partial \theta_y}{\partial y} + \frac{\theta_z}{R_y}\right)$$

$$\left(\varepsilon_{zz}\right)=\left(\frac{\partial w}{\partial z}\right)=\theta_z$$

$$\left(\gamma_{xy}\right)=\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{2w_0}{R_{xy}}\right)+z\left(\frac{\partial \theta_x}{\partial y}+\frac{\partial \theta_y}{\partial x}+\frac{2\theta_z}{R_{xy}}\right)$$

$$\left(\gamma_{yz}\right)=\left(\theta_y+\frac{\partial w_0}{\partial y}-\frac{v_0}{R_y}\right)+z\left(\frac{\partial \theta_z}{\partial y}-\frac{\theta_y}{R_y}\right)$$

$$\left(\gamma_{zx}\right)=\left(\theta_x+\frac{\partial w_0}{\partial x}-\frac{u_0}{R_x}\right)+z\left(\frac{\partial \theta_z}{\partial x}-\frac{\theta_x}{R_x}\right)$$

Strain written in matrix form as shown below

$$\{\varepsilon\}=\left\{\begin{matrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{zx}\end{matrix}\right\}_l=\left\{\begin{matrix}\varepsilon_{xx}^0\\ \varepsilon_{yy}^0\\ \varepsilon_{zz}^0\\ \gamma_{xy}^0\\ \gamma_{yz}^0\\ \gamma_{zx}^0\end{matrix}\right\}+z\left\{\begin{matrix}K_x^1\\ K_y^1\\ K_z^1\\ K_{xy}^1\\ K_{yz}^1\\ K_{zx}^1\end{matrix}\right\}$$

where,

$$\varepsilon_{xx}^0=\frac{\partial u_0}{\partial x}+\frac{w_0}{R_x},\ \varepsilon_{yy}^0=\frac{\partial v_0}{\partial y}+\frac{w_0}{R_y},\ \varepsilon_{zz}^0=\theta_z$$

$$\gamma_{xy}^0=\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{2w_0}{R_{xy}},\ \gamma_{yz}^0=\theta_y+\frac{\partial w_0}{\partial y}-\frac{v_0}{R_y},\ \gamma_{zx}^0=\theta_x+\frac{\partial w_0}{\partial x}-\frac{u_0}{R_x}$$

$$K_x^1=\frac{\partial \theta_x}{\partial x}+\frac{\theta_z}{R_x},\ K_y^1=\frac{\partial \theta_y}{\partial y}+\frac{\theta_z}{R_y},\ K_z^1=0$$

$$K_{xy}^1=\frac{\partial \theta_x}{\partial y}+\frac{\partial \theta_y}{\partial x}+\frac{2\theta_z}{R_{xy}},\ K_{yz}^1=\frac{\partial \theta_z}{\partial y}-\frac{\theta_y}{R_y},\ K_{zx}^1=\frac{\partial \theta_z}{\partial x}-\frac{\theta_x}{R_x}$$

Stress-strain relationship for lamina given as

$$\{\sigma\} = \left[\overline{Q_{ij}}\right]\{\varepsilon\} \tag{3.4}$$

where, $\{\sigma\}$ is the stress tensor, $\left[\overline{Q_{ij}}\right]$ is the reduced stiffness matrix and $\{\varepsilon\}$ is the strain tensor

Stress tensor can be modified into the force form as given below

$$\{F\} = [D]\{\varepsilon\} \tag{3.5}$$

where $[D]$ is the property matrix

3.3 Finite element modelling

For the analysis of composite laminate shell/plate structure finite element model has been proved to be suitable technique. The developed finite element model discretized using eight node isoperimetric quadrilateral Lagrangian elements with six degree of freedom per node. In the mid plane for any model the displacement vector *'d'* at any point can be taken as

$$d = \sum_{i=1}^{n} N_i(x, y) d_i \tag{3.6}$$

where, $\{d_i\} = \{u_{0i}, v_{0i}, w_{0i}, \theta_{xi}, \theta_{yi}, \theta_{zi}\}^T$ are the nodal displacement vectors and N_i is the corresponding interpolating function for $'i^{th}'$ node.

The mid plane strain vector in the form of nodal displacement vector is written as follows

$$\{\varepsilon\}_i = [B_i]\{d\} \tag{3.7}$$

where, $[B_i]$ is the strain displacement matrix

Non mechanical in-plane stress and moment resultants are written as below

$$\begin{aligned}\left\{N_x^N \quad N_y^N \quad N_{xy}^N\right\}^T &= \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} \left(\overline{Q_{ij}}\right)_k \{e\}_k \, dz \\ \left\{M_x^N \quad M_y^N \quad M_{xy}^N\right\}^T &= \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} \left(\overline{Q_{ij}}\right)_k \{e\}_k \, z \, dz,\end{aligned} \tag{3.8}$$

where, $\{e\}_k = \{e_x \quad e_y \quad e_{xy}\}_k^T$ is the non-mechanical strain oriented at an arbitrary angle θ_k of the *k*th lamina and is expressed as

$$\begin{Bmatrix} e_x \\ e_y \\ e_{xy} \end{Bmatrix}_k = \begin{bmatrix} m^2 & n^2 \\ n^2 & m^2 \\ -2mn & 2mn \end{bmatrix} \begin{Bmatrix} \beta_1 \\ \beta_2 \end{Bmatrix} (\Delta C) + \begin{bmatrix} m^2 & n^2 \\ n^2 & m^2 \\ -2mn & 2mn \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \end{Bmatrix} (\Delta T) \tag{3.9}$$

where, m, n are $\cos\theta_k, \sin\theta_k$, α_1, α_2 are the thermal coefficients and β_1, β_2 are the moisture coefficients of the lamina. ΔT and ΔC are the change in temperature and moisture concentration from reference temperature and moisture content respectively.

3.4 Energy calculation

Global displacement vector can be written as follows in matrix form

$$\left\{\bar{\delta}\right\} = \begin{Bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{Bmatrix} = [f]\{\delta\} \tag{3.10}$$

where, $[f]$ is the thickness coordinate function

The total strain energy (U) of composite can be expressed as

$$U = \frac{1}{2} \iint \left(\{\delta\}_l^T [B_L]_i^T [D][B_L]_i \{\delta\}_i \right) \tag{3.11}$$

where, $[D] = \int_{-h/2}^{+h/2} [T]^T \left[Q_{ij}\right][T] dz$

The total kinetic energy (T) of a composite can be expressed as

$$T = \frac{1}{2}\int_V \rho \left\{\dot{\delta}\right\}^T \left\{\dot{\delta}\right\} dV$$

where, $\left\{\dot{\delta}\right\}$ and ρ are the first order differential of the displacement vector with respective to time and mass density respectively.

After substitution final form of kinetic energy can be written as

$$T = \int_A \left(\left[N_i\right]^T \left[m\right]\left[N_i\right] dA\right)\left\{\ddot{\delta}\right\} \tag{3.12}$$

The total work done (W) by an externally applied load (F) can be written as

$$W = \int_A \left\{\delta\right\}^T \left\{F\right\} dA \tag{3.13}$$

3.5 Governing Equations

For the free vibration of laminated shell the governing equation can be obtained by Hamilton's principle and it is written as follows

$$\delta\int_{t_1}^{t_2} \left(T - U\right) dt = 0 \tag{3.14}$$

Substituting the total kinetic energy (T) and total strain energy (U) into the equation (3.14), the final form of equation written as follows

$$\left[M\right]\left\{\ddot{d}_i\right\} + \left[K\right]\left\{d_i\right\} = 0 \tag{3.15}$$

where, $\ddot{d}_i$ is the acceleration , d_i is the displacement vector

Natural frequency and the system conceded by dropping the appropriate term to obtain the eigenvalue from of governing equation as follows

$$\left([K]-\omega^2[M]\right)\Delta = 0 \tag{3.16}$$

where, ω and Δ are the natural frequency and corresponding eigenvector.

3.6 Dynamic Equilibrium Equations

The assembly of element stiffness matrix, mass matrix, initial stress stiffness matrix and load vector neglecting the damping effect the global dynamic equilibrium can be written as

Free vibration equation

$$[M]\{\ddot{u}\} + \{[K]+[K_\sigma]\}\{u\} = 0 \tag{3.17}$$

Forced vibration equation

$$[M]\{\ddot{u}\} + \{[K]+[K_\sigma]\}\{u\} = \{F\} \tag{3.18}$$

where, $[M]$ is the global mass matrix, $[K]$ is the global stiffness matrix, $[K_\sigma]$ is the global residual stiffness matrix, $\{u\}$ and $\{\ddot{u}\}$ are global displacement and acceleration vector and $\{F\}$ is the global load vector.

Chapter 4

Experimental Analysis

4.1 Fabrication of composite laminate

There are different methods for the fabrication of the composite laminates. For the fabrication of Glass-Epoxy composite laminated cylindrical shell hand layup method is used. Hand layup method is the oldest and simplest open molding method for the fabrication of composites. This method is labor intensive and low volume method particularly suitable for large components, like boat hulls, reinforcing mat, glass.

Cylindrical shaped mould as required is prepared for the fabrication, a silicone gel is sprayed on to the release sheet and this sheet will be place on to the mould, this silicone gel will help for easy removal of composite from the release sheet, before poring hardener and epoxy we take epoxy and hardener with 10 percentage by weight of epoxy that is we take 270gm of epoxy and 27gm of hardener and we mix this solution continuously for five minutes. After mixing over the mixture is poured on to the already prepared mould as much required and level the epoxy mixture with scale, place the already prepared by required dimension glass fiber on to the first layer of epoxy mixture, again pour the epoxy mixture on to the glass fiber and by using roller remove the air bubbles and repeat the process till we get required number of layers. This process should be completed within half an hour of mixing because of exothermic reaction the epoxy mixture will be hardened. After achieving the required numbers of layers apply dead loads of 15 to 20 kg on to it and place it for 48 hours. Figure 4.1 shows the fabricated glass epoxy composite laminated cylindrical shell.

Figure 4.1 cylindrical composite shell

4.2 Evaluation of material property of composite laminate

For the free vibration analysis of composite laminate cylindrical shell material properties has to be determined experimentally. By using unidirectional tensile test machine (UTM) we can find Young’s modulus of composite laminate. For the Young’s modulus three pieces are cut in the transvers, longitudinal and at an inclined angle of 45 degrees to the longitudinal direction from the composite laminate which was modeled previously. The dimensions of the pieces are cut according to the ASTM standard (D 3039/ D 3039M). The cut sections are fixed in universal testing machine (UTM) INSTRON 1195 and the deformation was applied at the rate of 1 mm/minute as shown in the Figure 4.2. This load will exert tensile stress on cut pieces due to this the cut pieces will fail at some point. In this experiment stress vs. strain graph will be plotted from this plot we find the slope and the slope of the graph is nothing but Young’s modules. Similarly the graph is plotted for remaining cut pieces and the average value of the Young’s modulus is taken.

The Poisson's ratio is taken as 0.17 for the calculation purpose. To find the shear modulus of the different specimen the formula is given and by substituting the data value obtained from the experiment the shear modulus can be find out.

$$G_{1t} = \frac{1}{\frac{4}{E_{45}} - \frac{1}{E_l} - \frac{1}{E_t} - \frac{2\upsilon_{12}}{E_l}} \tag{3.18}$$

The material properties obtained from the experiment are as shown in the Table.4.1

Figure 4.2 Universal testing machine

Figure 4.3 Composite laminated cut specimen

Table.4.1 Geometrical & Material properties of Glass Epoxy composite laminate

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4.3 Experimental procedure

Free vibration of Glass-Epoxy composite laminate can be found out experimentally by using the NI-cDAQ 9178 (National Instrument compact data acquisition) device at NIT Rourkela. The composite laminated cylindrical shell is fixed in the fixture as cantilever support condition, the accelerometer is attached on the cylindrical shell and with the help of impact hammer the cylindrical shell is excited at any random points the accelerometer captures the vibration signals. The accelerometer is sensor which captures the vibration signals and it convert into the analogue voltage signal. The cDAQ device receives the analogue voltage signals from the accelerometer, this analogue voltage signal then converted into digital signal with the help of the analogue to digital converter which is inbuilt inside the cDAQ device, these digital signals then processed by the LAB-VIEW software. In the LAB-VIEW software a virtual instrument program circuit is developed and it help in supply of input parameters and it shows output parameters on computer screen. The block diagram of the LAB-VIEW software is as shown in the Figure 4.4 for the data acquisition (cDAQ) of the required signals and subsequent analysis. By using single and double integration blocks in the LAB-VIEW circuit from the recorded acceleration the velocity and displacement of the cylindrical shell can be calculated. The peak of the frequency response spectrum for different modes gives the natural frequencies of the vibration.

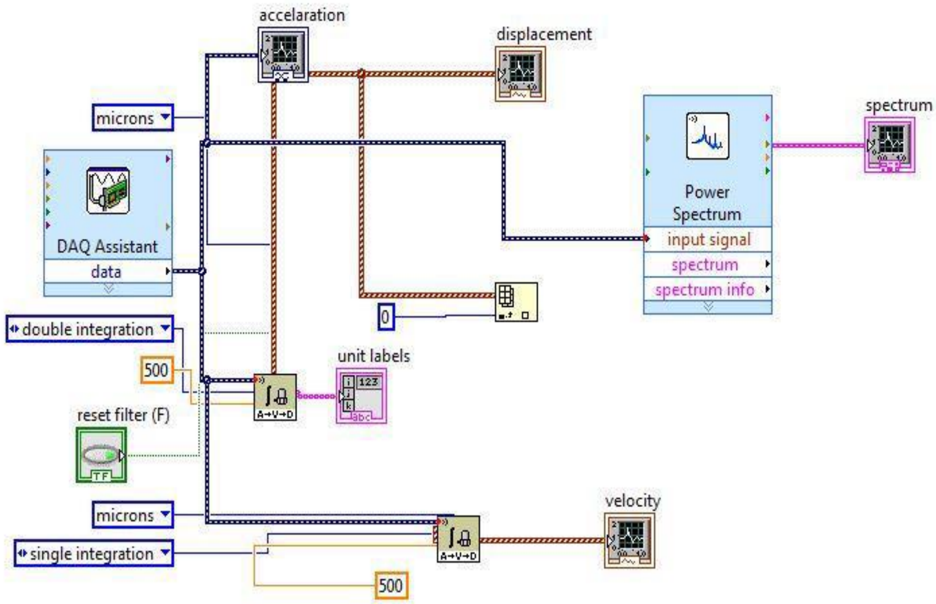

Figure 4.4 LAB-VIEW circuit diagram for computer interface

The first peak response of the frequency for eight layered Glass-Epoxy composite laminated cylindrical shell is given as shown in the Figure 5.5 (a)-(d) which is the first natural frequency of the cylindrical shell. These experimental results are compared with the numerical result using simulation model.

Chapter 5

Results and Discussion

Convergence study is performed by computing the vibration responses of cylindrical shell panel with different mesh refinement. In order to check the validity of the present model some examples have been solved and compared with available published literature. In addition to that transient response also validated with available published literature. Some of the examples also solved in this work.

5.1 Convergence behaviour of composite laminated cylindrical shell

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Table.5.1 Natural frequency of composite cylindrical shell for different mesh refinement

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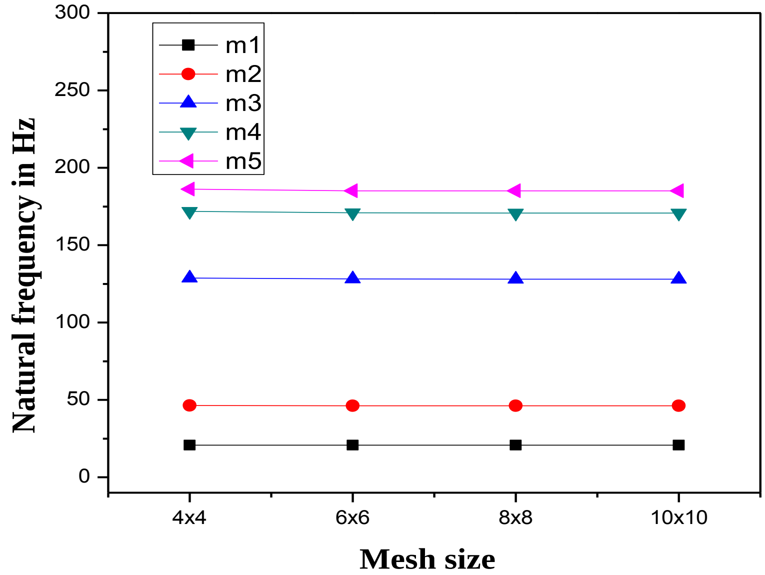

Figure 5.1 Convergence of composite laminated cylindrical shell in cantilever support condition

5.2 Comparison Study for Free Vibration

Natural frequencies of composite cylindrical shell, spherical shell and composite laminate plate are compared with available published literature. For this comparison study composite plate with different moisture content is taken and natural frequency for each case is computed. These results are validated with present result using ANSYS APDL software. Also experimental results of composite cylindrical shell are computed and compared with ANSYS result.

5.2.1 Comparison study of cylindrical shell under different moisture content

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Table.5.2 Natural frequency of laminated composite cylindrical shell under different moisture content

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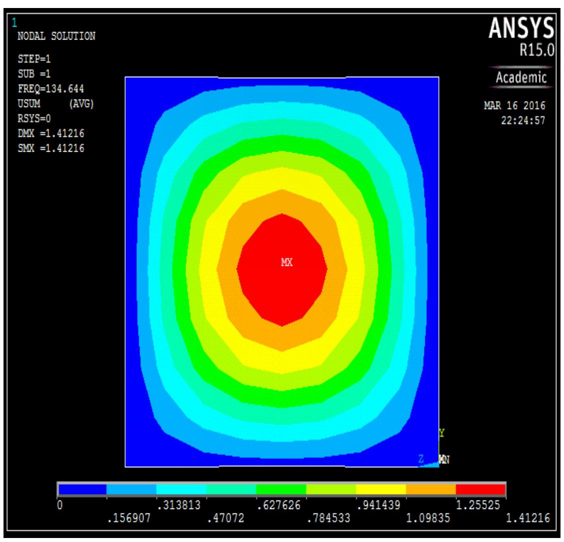

Figure 5.2 (a) Natural frequency for = 0

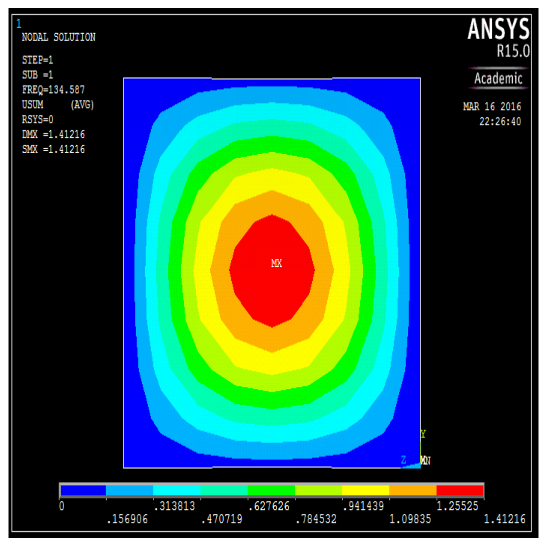

Figure 5.2 (b) Natural frequency for = 0.25

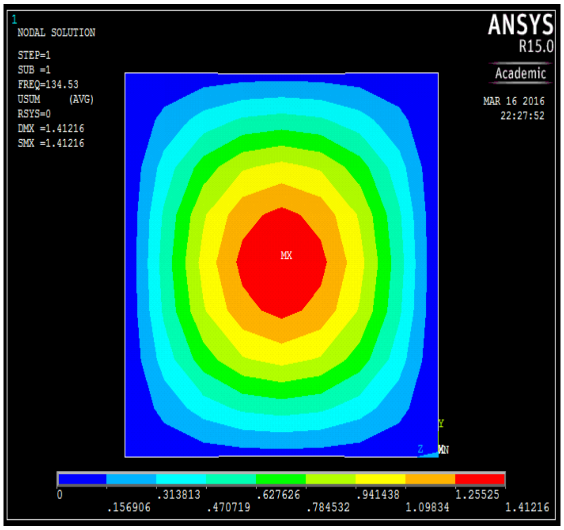

Figure 5.2 (c) Natural frequency for ΔC = 0.5

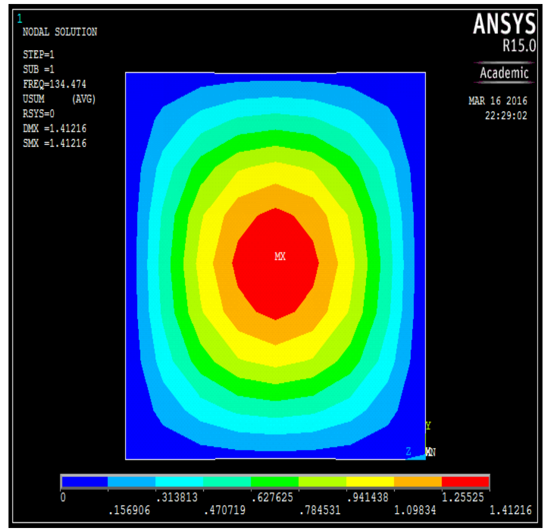

Figure 5.2 (d) Natural frequency for ΔC = 0.75

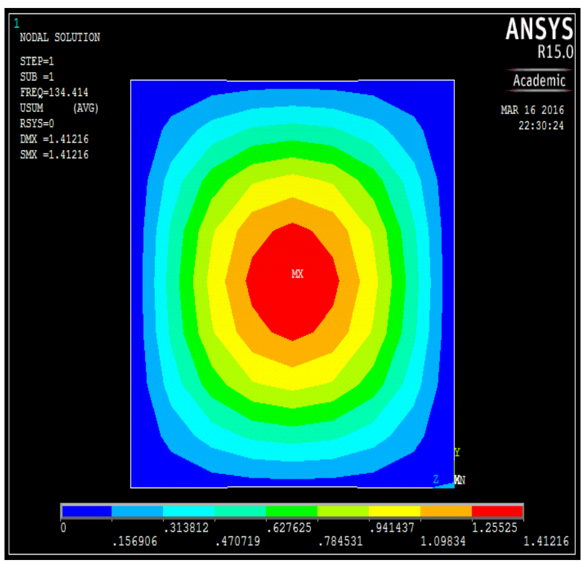

Figure 5.2 (e) Natural frequency for = 1.0

5.2.2 Comparison study of spherical shell under different moisture content

Table.5.3, the natural frequency of composite laminate of spherical shell under different moistur concentration ΔC = 0, 0.25, 0.5, 0.75 and 1.0 as shown are computed and this results are validated with reference [4]. Figure 5.3 (a)-(d) shows the natural frequency of laminated composite spherical shell under different moisture concentration.

Table.5.3 Natural frequency of laminated composite spherical shell under different moisture content

Moisture content	Parhi et al. [4] (*Hz*)	Present (*Hz*)
0.00	202.02	206.19
0.25	201.91	206.14
0.50	201.82	206.10
0.75	201.72	206.06
1.00	201.64	206.01

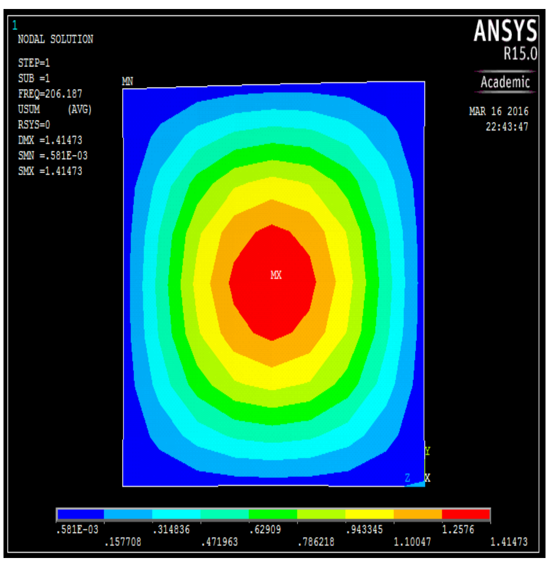

Figure 5.3 (a) Natural frequency for = 0

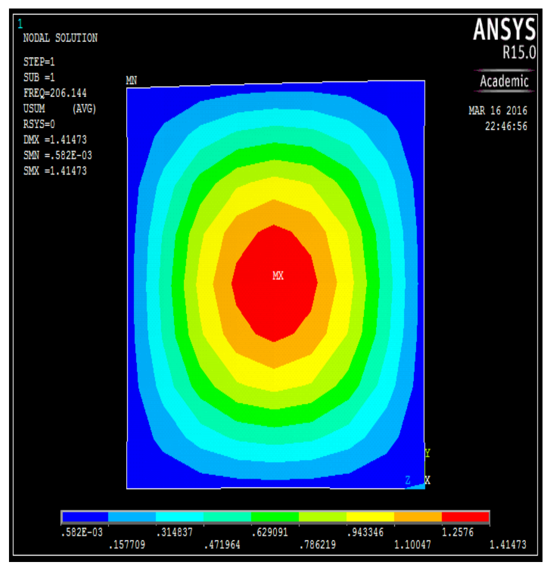

Figure 5.3 (b) Natural frequency for = 0.25

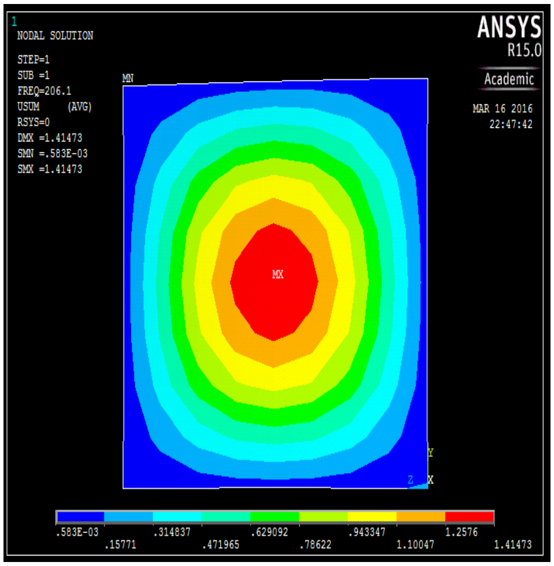

Figure 5.3 (c) Natural frequency for = 0.5

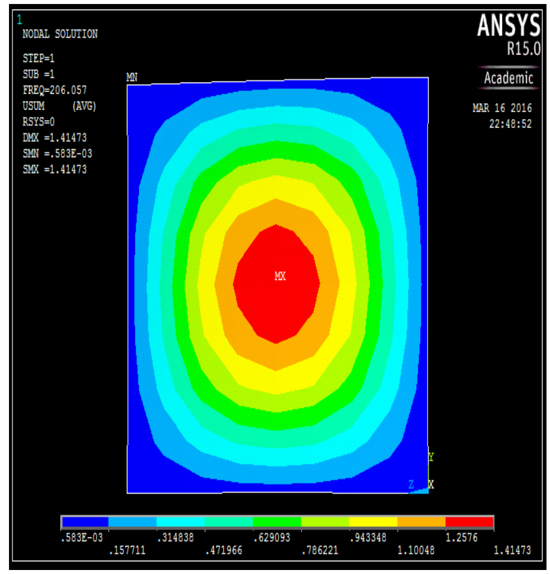

Figure 5.3 (d) Natural frequency for = 0.75

5.2.3 Comparison study of composite plate under different support condition

In Table 5.4, for 8-layered composite plate $(0/90/45/90)_s$ natural frequencies for cantilever (CFFF), four side clamped (CCCC) and four side simply supported (SSSS) case are computed using ANSYS APDL software as shown and these results are validated with reference [13]. Figure 5.4 (a)-(c) shows the natural frequency for different support conditions like CFFF, CCCC, SSSS.

Table.5.4 Natural frequency for laminated composite plate under different support condition

Support condition	Ju & Lee [13] (*Hz*)	Present (*Hz*)
CFFF	41.347	41.167
CCCC	346.59	343.23
SS,SS	164.37	163.53

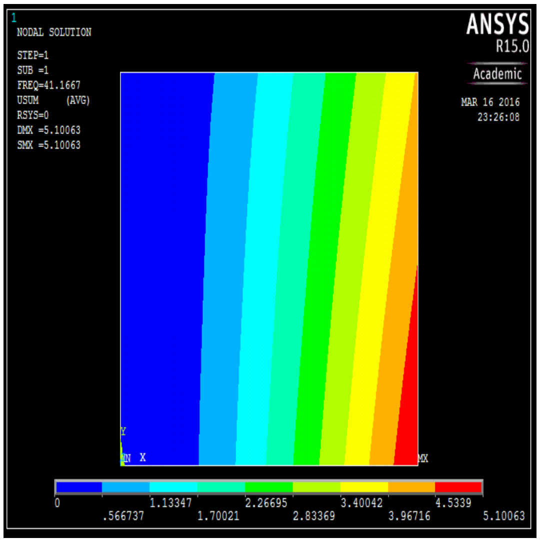

Figure 5.4 (a) Natural frequency of composite plate plate for cantilever support condition

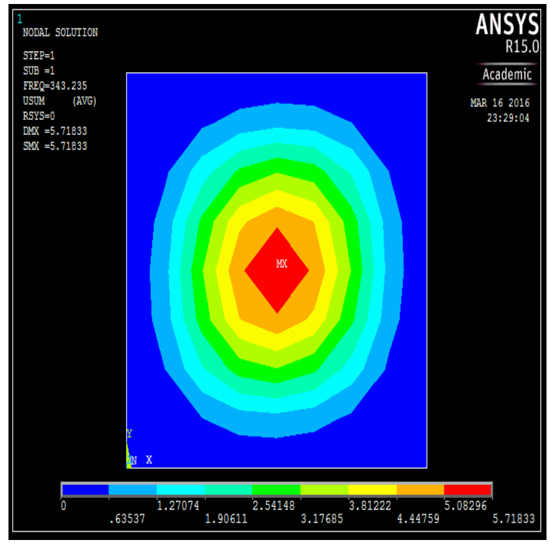

Figure 5.5 (b) Natural frequency of composite for four side clamped condition

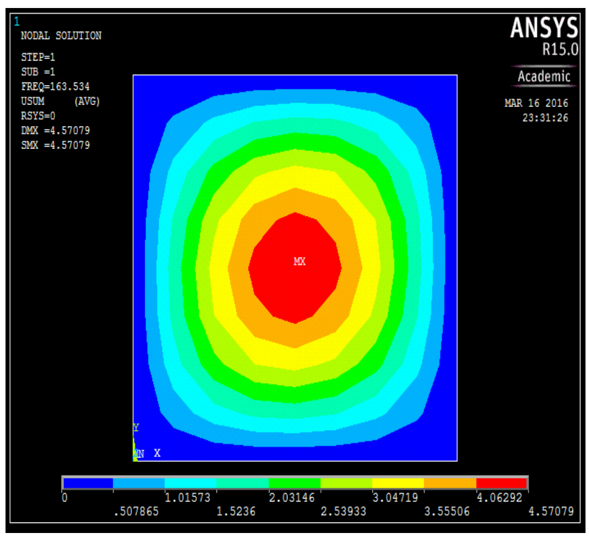

Figure 5.4 (c) Natural frequency of composite plate for four sides simply supported condition

5.3 Experimental results

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Table.5.5 Experimental natural frequency of laminate composite cylindrical shell panel for different aspect ratio

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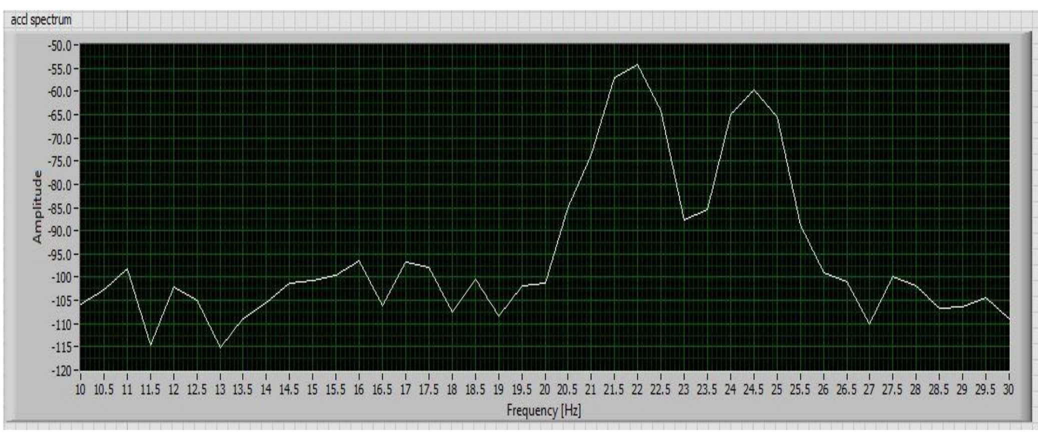

Figure 5.5 (a) Natural frequency (22 *Hz*) for cantilever support condition for aspect ratio 1.2

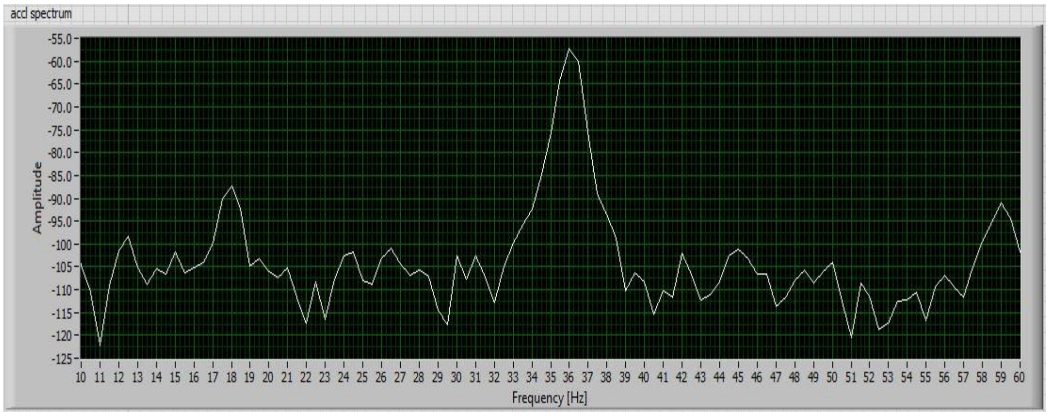

Figure 5.5 (b) Natural frequency (36 *Hz*) for cantilever support condition for aspect ratio 1.5

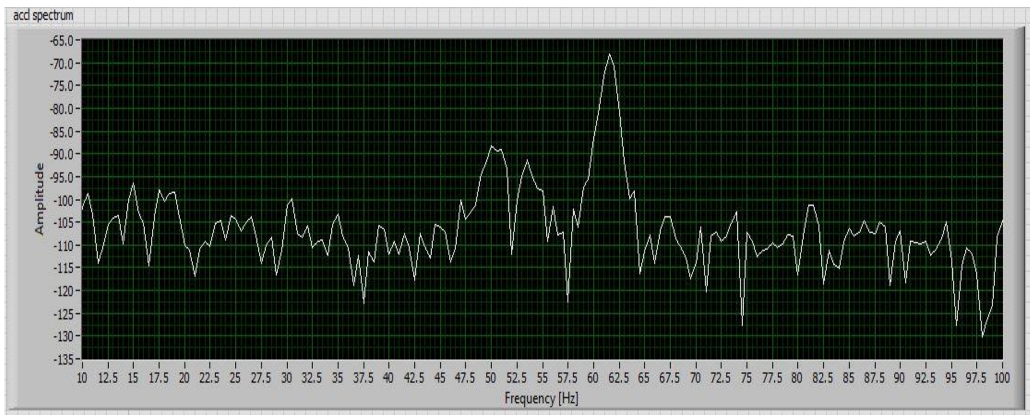

Figure 5.5 (c) Natural frequency (66 *Hz*) for cantilever support condition for aspect ratio 2.0

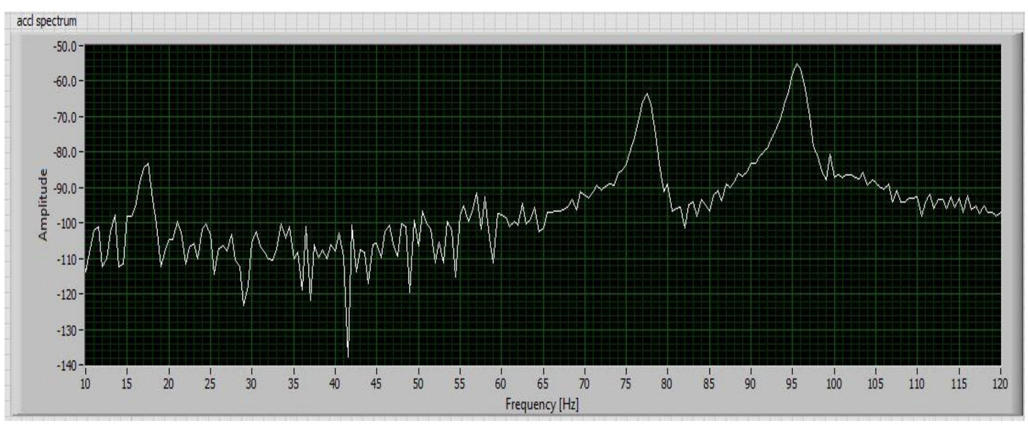

Figure 5.5.d Natural frequency (96 *Hz*) for cantilever support condition for aspect ratio 2.5

5.4 Numerical Illustration

5.4.1 Effect of aspect ratio on free vibration of composite laminate

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Table.5.6 Natural frequency of laminated composite cylindrical shell panel for different aspect ratio

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Figure 5.6 Natural frequency graph of laminated composite cylindrical shell panel for different aspect ratio

5.4.2 Effect of thickness ratio on free vibration of composite laminate

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Table.5.7 Natural frequency of laminated composite cylindrical shell panel for different thickness ratio

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Thickness Ratio

Figure 5.7 Natural frequency of laminated composite cylindrical shell panel with different thickness ratio

5.4.3 Effect of different support condition on free vibration of composite laminate

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Table.5.8 Natural frequency of laminated composite cylindrical shell panel for different support condition

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Figure 5.8 Natural frequency of the laminated composite cylindrical shell panel for different support condition

5.5 Transient Analyses

5.5.1 Validation of non-dimensional transient response of the composite laminate plate

In this present work transient response of a two layered laminated composite square plate are computed. The material properties and geometrical parameters are tabulated in the Table.5.9 are considered for the analysis. The non-dimensional displacement is computed for time step of $0.05 \mu s$ under uniform step load of q_0. Figure 5.9 shown below presents the non-dimensional deflection of composite laminate plate under uniform loading in transient analysis and these result are validated with Maleki [20] et al. and Reddy [21]. From the Figure 5.9 it is clearly seen that the results are in very much good agreement with references.

The non-dimensional displacement of composite for transient analysis can be calculated by using formula given below

$$\bar{w} = 100w\left[\frac{E_2 h^3}{q_0 a^4}\right]$$

where, $\bar{w}$ is the non-dimensional displacement, w displacement, q_0 uniform step loading

Table.5.9 Material and geometrical properties of two layered laminated composite square plate

a	25 cm
h	1 cm
E_1	52.5×10^6 N/cm^2
$E_2 = E_3$	2.1×10^6 N/cm^2
$G_{12} = G_{13}$	1.05×10^6 N/cm^2
G_{23}	0.42×10^6 N/cm^2
$\nu_{12} = \nu_{23} = \nu_{31}$	0.25

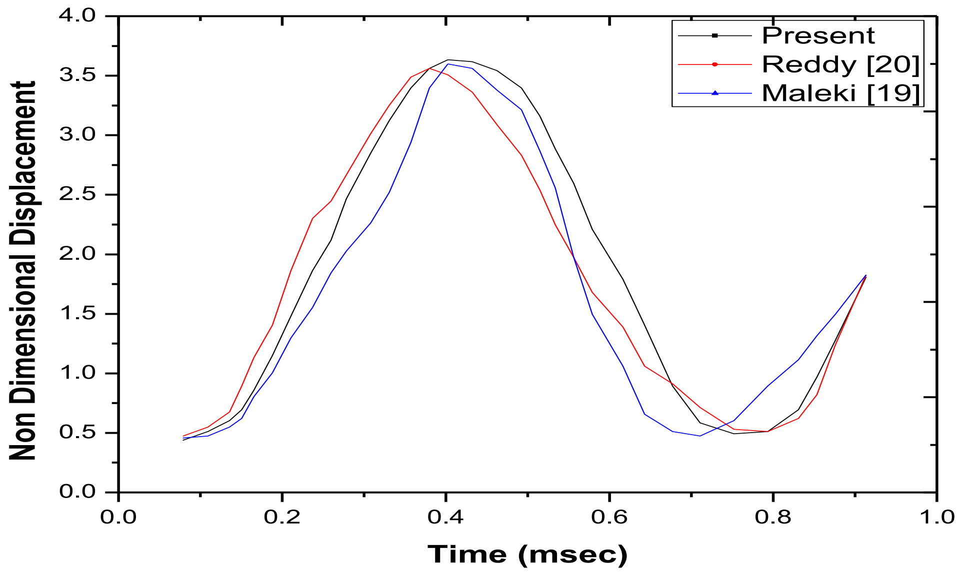

Figure 5.9 Non Dimensional Displacement of two layered composite plate for simply supported condition

5.6 Numerical Illustration

5.6.1 Effect of the aspect ratio on the transient response of composite laminate cylindrical shell

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Figure 5.10 Effect of aspect ratio on transient responses of composite cylindrical shell panel for simply supported condition under uniform step load

5.6.2 Effect of the thickness ratio on the transient response of composite laminate cylindrical shell

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Figure 5.11 Effect of thickness ratio on transient response of a laminated cylindrical shell panel under simply supported boundary condition for uniform step load

Chapter 6

Clouser

6.1 Concluding Remarks

The free vibration and transient response of laminated composite cylindrical panel are studi ed. Free vibration behavior of the laminated composite cylindrical and spherical shell panels are validated with the results of the published literature under hygrothermal condition. Also, the transient response of the two layer laminated plate for step loading is validated by comparing the responses with results of the available published literature. 8-layered composite laminate cylindrical shell is fabricated using hand layup method and material properties are found out experimentally using UTM INSTRON 1195. Experimentally free vibration response of 8-layered composite laminated cylindrical panel is computed and validated with simulation result generated using ANSYS. Free vibration behavior and transient response under different parameters like different thickness ratio, aspect ratio, and different boundary condition studied and discussed in detail.

- From the above study, we can conclude that as the aspect ratio of the laminate composite cylindrical shell panel increases the natural frequency of the laminated cylindrical shell panel increases because stiffness increases.

- As the thickness of the laminated composite cylindrical shell panel increases the natural frequency of the laminate cylindrical shell panel decreases because stiffness decreases.

- As the constrained degree of freedom of composite laminated cylindrical shell panel increases natural frequency of the cylindrical shell panel increases.

- In the transient analysis case as the aspect ratio of the cylindrical shell panel increases the displacement of the cylindrical shell panel decreases

- In the transient analysis as the thickness ratio of the cylindrical shell panel increases the displacement of the cylindrical shell panel increases.

6.2 Future Scope of the Research

- The present work is done for linear analysis case only; this work can beeextended to non-linear analysis of the laminated composite panel.
- The present work can be extended for the analysis of different type of the shell panels.
- The present work can be extended to examine the linear and non-linear forced/damped vibration under aerodynamic and hydraulic loading conditions.
- The present work can be extended to examine the free/forced vibration and acoustic analysis of laminated composite structure under different delamination conditions.

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Dissemination

Article under preparation

R.Srikanth, C.K.Hirwani, S.K Panda "Experimental & Numerical Analysis of Free Vibration &Dynamic behaviour of Laminated Composite Shallow Shell under Hygrothermal Condition"